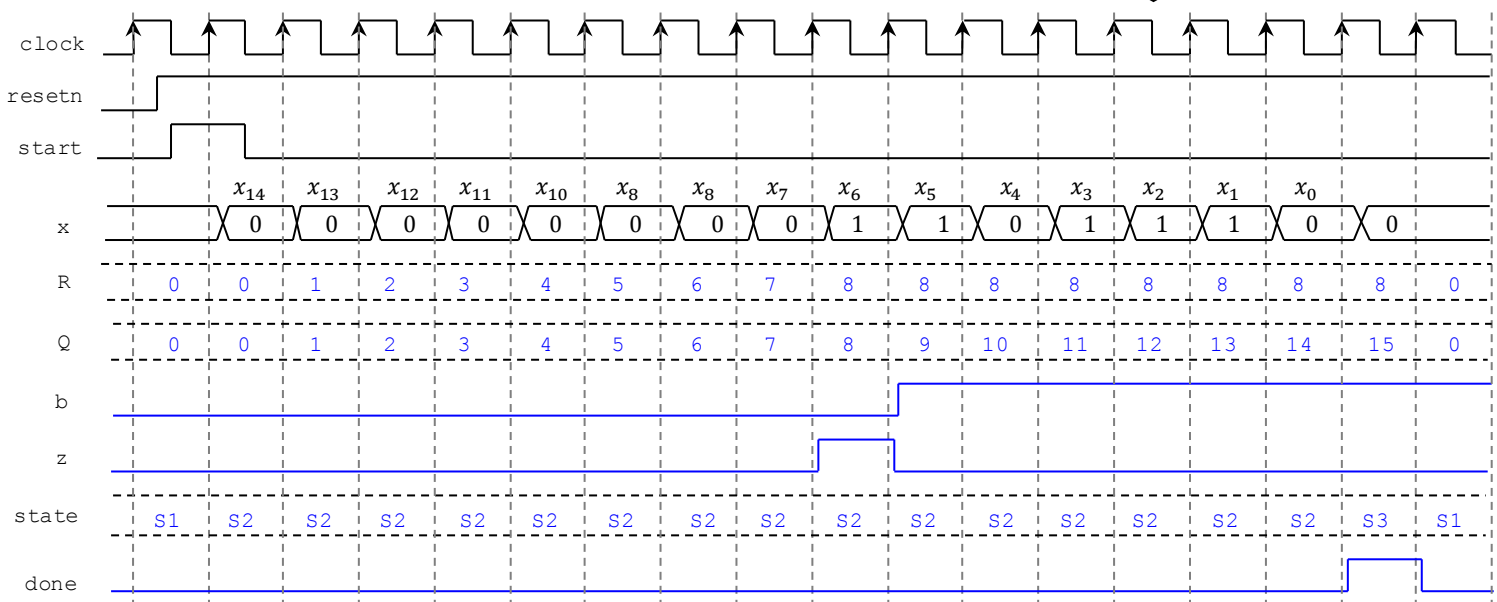
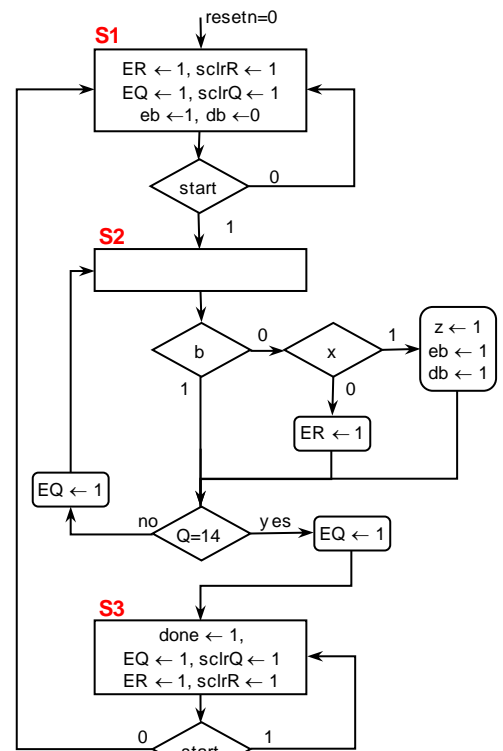


(Due date: January 30<sup>th</sup> @ 7:30 pm)

### PROBLEM 1 (10 PTS)

- 

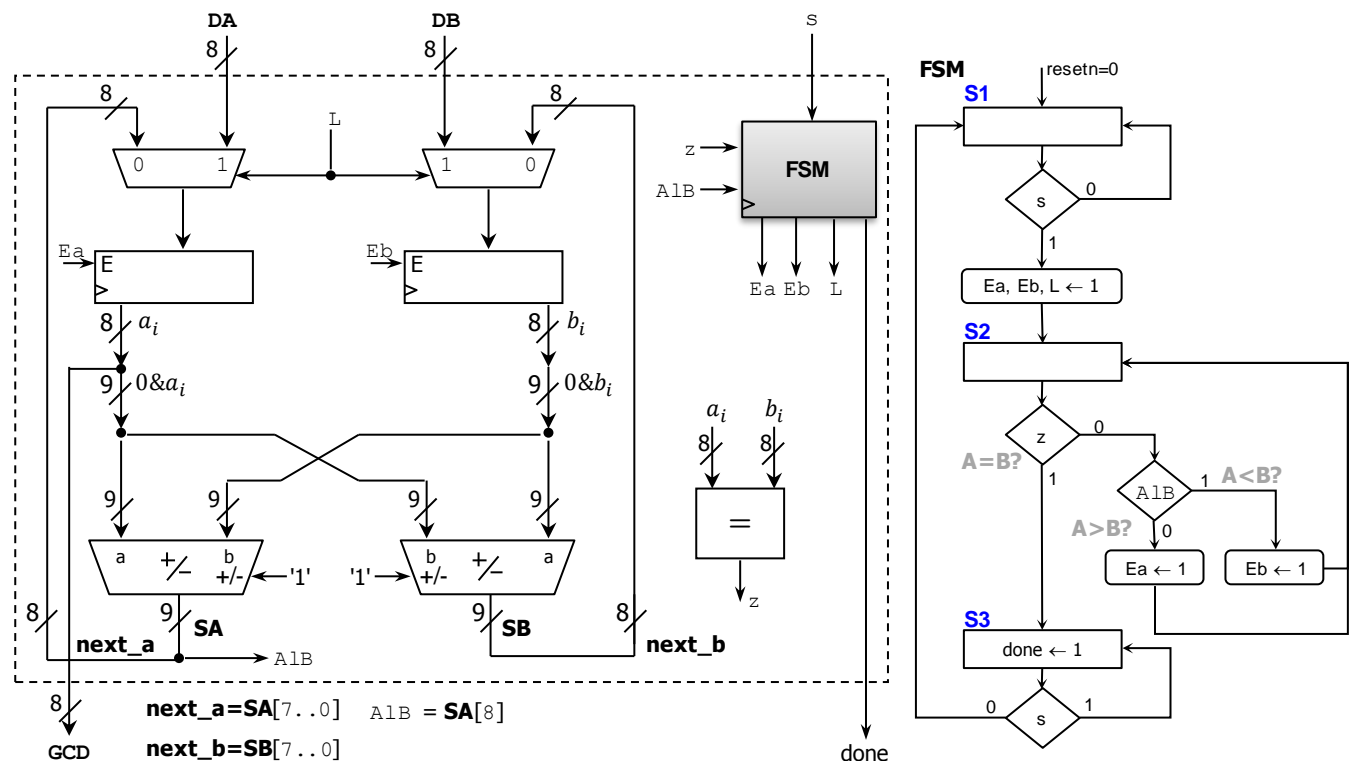


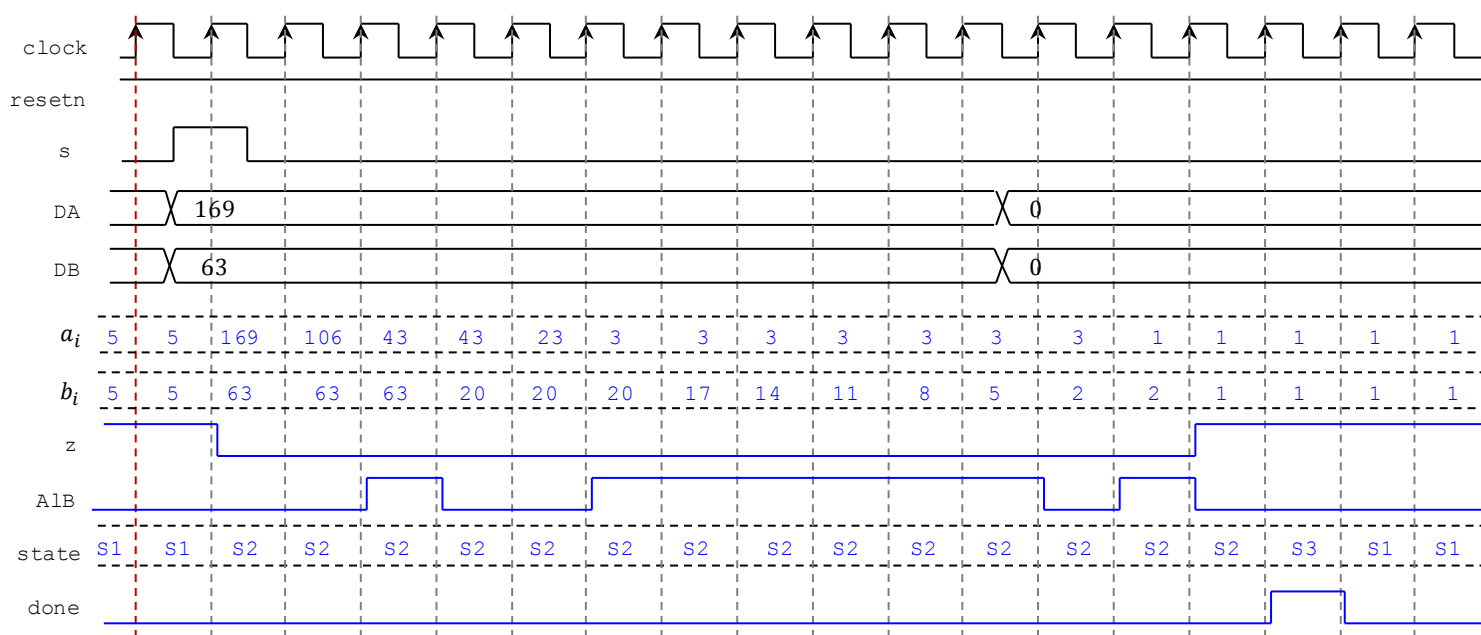
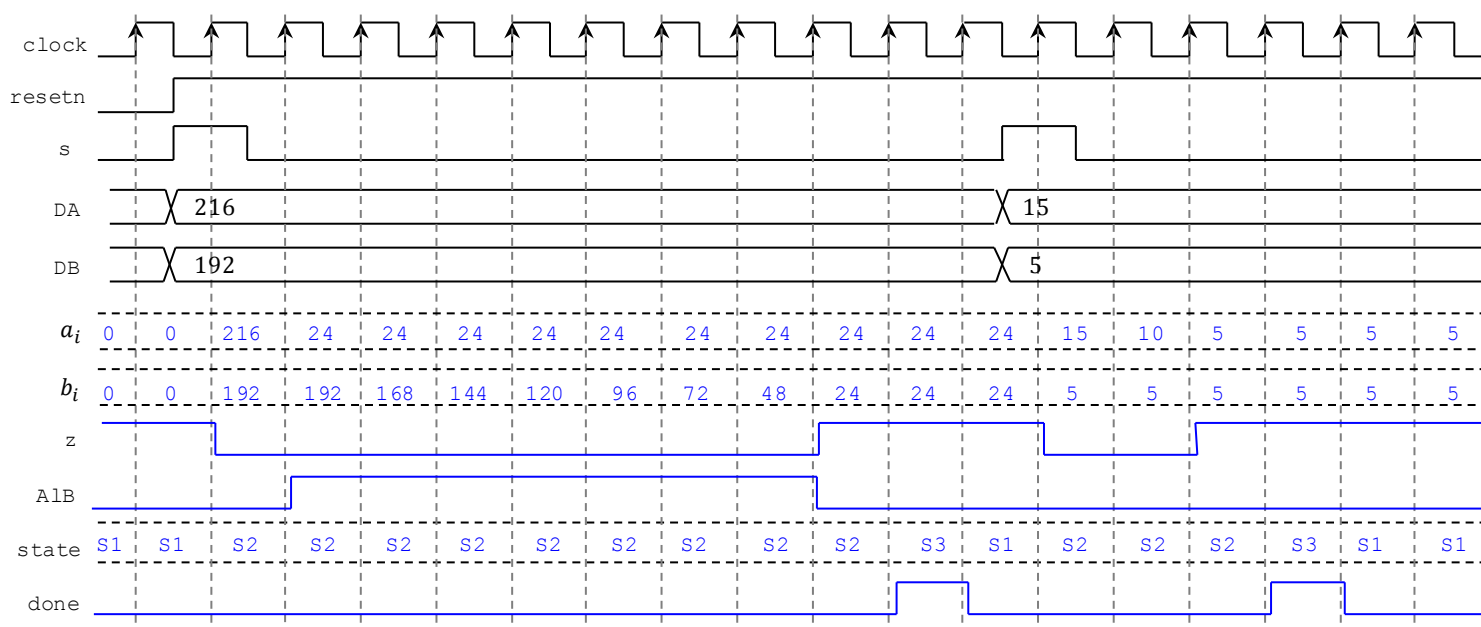
## PROBLEM 2 (40 PTS)

- **Greatest Common Divisor (GCD):** This iterative circuit processes two 8-bit unsigned numbers (A, B) and generates the GCD of A and B. For example:
  - ✓ If  $A = 216, B = 192 \rightarrow \text{GCD} = 24$ .
  - ✓ If  $A = 132, B = 72 \rightarrow \text{GCD} = 12$ .
  - ✓ If  $A = 169, B = 63 \rightarrow \text{GCD} = 1$ .
- The circuit is based on the Euclid's GCD Algorithm:

```
a,b: unsigned integers
while a ≠ b
  if a > b
    a ← a-b
  else
    b ← b-a
  end
end
return a
```
- The figure depicts the (in ASM form) and a datapath circuit.  
Input data: DA, DB. Output data: GCD.
- ✓ Complete the timing diagram of the digital circuit (next page). Note that 3 pairs of numbers are evaluated.
- ✓ Write a structural VHDL code. You MUST create (or re-use) a file for i) N-bit register, ii) Adder/Subtractor, iii) Bus MUX 2 to 1, iv) Finite State Machine, and v) Top file (where you will interconnect all the components).
- ✓ Write a testbench according to the timing diagram shown (next page). Simulate the circuit (Behavioral simulation). Verify that the simulation is correct by comparing it with the timing diagram you completed manually.
- ✓ Upload (as a .zip file) the following files to Moodle (an assignment will be created):
  - VHDL code files
  - VHDL testbench

See attached .zip file: SolutionsHW1p2.zip





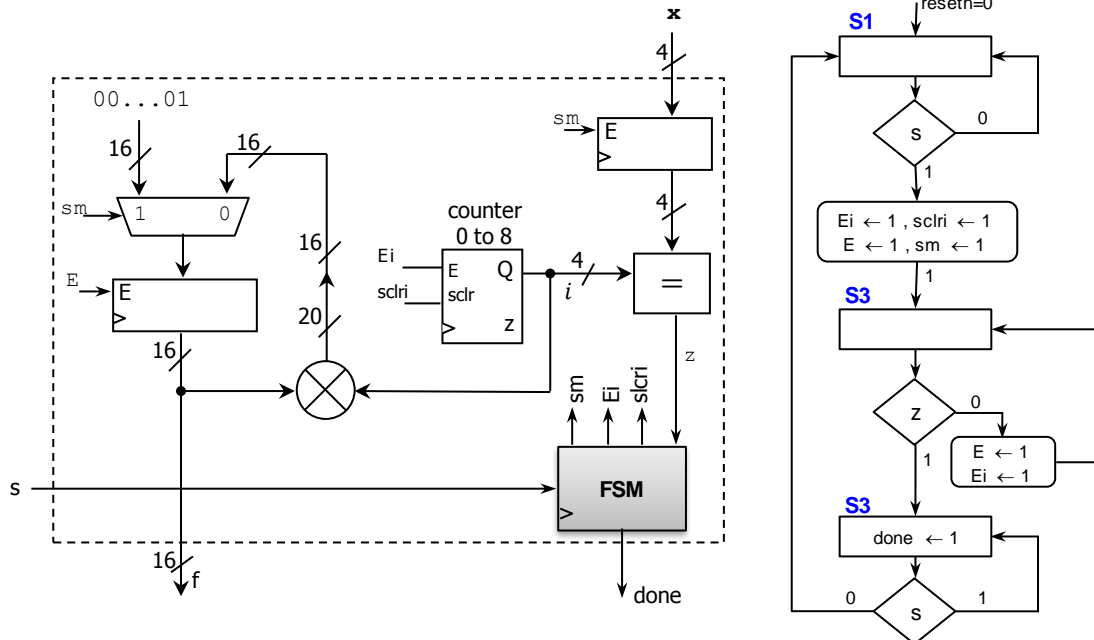
### PROBLEM 3 (25 PTS)

- **Factorial Computation:** The following algorithm computes the factorial of an unsigned number.

```
x: unsigned integer
f = 1
if x ≠ 0
    for i = 1 to x
        f ← fxi
    end
end
return f
```



- We want to design a circuit that reads in an unsigned number  $x$  and generates  $f = x!$ .
- Operation: The circuit reads data in when the  $s$  signal (usually a one-cycle pulse) is asserted. When the result is ready, the signal  $done$  is asserted.
  - ✓ Inputs:  $x$  (input data),  $s$  (start signal).
  - ✓ Outputs:  $f$  (factorial),  $done$ .
  - ✓ We restrict the bitwidth of  $f$  to 16 bits. As a result, the largest  $x$  is 8 (as  $8! = 40320$ ).
- Sketch the circuit: FSM + Datapath components. Specify all the I/Os of the FSM, as well as the signals connecting the FSM and the Datapath components (as in Problem 2).
  - ✓ You can use an array multiplier as a component. This multiplier will multiply two values:  $f$  and  $i$  (as per the algorithm).  
Note: the multiplication only needs 16 bits (even if the sum of the input bitwidths is greater than 16 bits). Thus, some MSBs will need to be discarded (not a problem since the largest  $f$  fits with 16 bits).
  - ✓ Feel free to use any other standard component (e.g. counter, register, comparator, busmux).
  - ✓ Your circuit should compute any factorials from  $x = 0$  to  $x = 8$ .
  - ✓ Provide the State Diagram (in ASM form) of the FSM.



# PROBLEM 4 (15 PTS)

- Calculate the result of the following operations, where the operands are signed integers. For the division, calculate both the quotient and the residue. **No procedure = zero points.**

10011 × 11011	10010 × 01001	100101 ÷ 1101	01111010 ÷ 100	100010 ÷ 0101
------------------	------------------	------------------	-------------------	------------------

$$\begin{array}{r}
 10011 \times \\
 11011 \\
 \hline
 01101 \\
 00101 \\
 \hline
 11011 \\
 00000 \\
 11011 \\
 \hline
 10000001 \\
 \hline
 01000001
 \end{array}$$

$$\begin{array}{r}
 10010 \times \\
 01001 \\
 \hline
 01110 \times \\
 01001 \\
 \hline
 1110 \times \\
 1001 \\
 \hline
 1110 \\
 1110 \\
 \hline
 1111110 \\
 \hline
 01111110 \\
 \hline
 10000010
 \end{array}$$

$$\checkmark \frac{100101}{1101} = \frac{-27}{-3}$$

$$\begin{array}{r}
 01001 \\
 11 \overline{) 11011} \\
 \underline{11} \phantom{000} \\
 0011 \\
 \underline{11} \\
 0
 \end{array}$$

$$\text{To unsigned: } \frac{011011}{011}$$

$$\text{Unsigned Integer Division: } Q' = 1010, R' = 0 \\
 \rightarrow Q = Q' = 01001, \rightarrow R = -R' = 2C(0) = 0$$

$$\text{Verification: } -27 = (-9 \times -3) + 0$$

$$\checkmark \frac{01111010}{100} = \frac{122}{-4}$$

$$\begin{array}{r}
 0011110 \\
 100 \overline{) 1111010} \\
 \underline{100} \phantom{000} \\
 111 \phantom{00} \\
 \underline{100} \phantom{00} \\
 110 \phantom{00} \\
 \underline{100} \phantom{00} \\
 101 \phantom{00} \\
 \underline{100} \phantom{00} \\
 10
 \end{array}$$

$$\text{To unsigned: } \frac{01111010}{0100}$$

$$\text{Unsigned Integer Division: } Q' = 11110, R' = 10 \\
 \rightarrow Q = -Q' = 2C(011110) = 100010, \rightarrow R = R' = 010$$

$$\text{Verification: } 122 = (-30 \times -4) + 2$$

$$\checkmark \frac{100010}{0101} = \frac{-30}{5}$$

$$\begin{array}{r}
 00110 \\
 101 \overline{) 11110} \\
 \underline{101} \phantom{00} \\
 101 \phantom{00} \\
 \underline{101} \phantom{00} \\
 00
 \end{array}$$

$$\text{To unsigned: } \frac{011110}{0101}$$

$$\text{Unsigned Integer Division: } Q' = 110, R' = 0 \\
 \rightarrow Q = -Q' = 2C(0110) = 1010, \rightarrow R = -R' = 2C(0) = 0$$

$$\text{Verification: } -30 = (-6 \times 5) + 0$$

## PROBLEM 5 (10 PTS)

- Compute the result of the additions and subtractions for the following fixed-point numbers.

UNSIGNED (1 pt. each)		SIGNED	
0.101010 + 1.0110101	1.00101 - 0.0000111	10.001 + 1.001101	0.0101 - 1.0101101
	100.1 + 0.1000101	1000.0101 - 111.01001	101.0001 + 1.0111101

### UNSIGNED:

$$\begin{array}{r}
 c_8=1 \\
 c_7=1 \\
 c_6=1 \\
 c_5=1 \\
 c_4=0 \\
 c_3=1 \\
 c_2=0 \\
 c_1=0 \\
 c_0=0 \\
 \hline
 0.101010 + \\
 1.0110101 \\
 \hline
 1.0000011
 \end{array}$$

$$\begin{array}{r}
 b_7=0 \\
 b_6=0 \\
 b_5=0 \\
 b_4=1 \\
 b_3=1 \\
 b_2=1 \\
 b_1=1 \\
 b_0=0 \\
 \hline
 1.00101 - \\
 0.0000111 \\
 \hline
 1.0000000
 \end{array}$$

$$\begin{array}{r}
 c_{10}=0 \\
 c_9=0 \\
 c_8=0 \\
 c_7=1 \\
 c_6=0 \\
 c_5=0 \\
 c_4=0 \\
 c_3=0 \\
 c_2=0 \\
 c_1=0 \\
 c_0=0 \\
 \hline
 10.001 + \\
 1.001101 \\
 \hline
 11.010101
 \end{array}$$

### SIGNED:

$$\begin{array}{r}
 c_9=1 \\
 c_8=1 \\
 c_7=0 \\
 c_6=0 \\
 c_5=0 \\
 c_4=1 \\
 c_3=0 \\
 c_2=0 \\
 c_1=0 \\
 c_0=0 \\
 \hline
 11.00001 + \\
 11.00001 \\
 \hline
 10.00002
 \end{array}$$

$$\begin{array}{r}
 0.0101 - \\
 1.0101101 \\
 \hline
 0.99989
 \end{array}$$



$$\begin{array}{r}
 c_8=0 \\
 c_7=0 \\
 c_6=0 \\
 c_5=0 \\
 c_4=0 \\
 c_3=0 \\
 c_2=0 \\
 c_1=0 \\
 c_0=0 \\
 \hline
 0.0101 - \\
 1.0101101 \\
 \hline
 0.99989
 \end{array}$$

$$\begin{array}{r}
 1000.0101 - \\
 111.01001 \\
 \hline
 889.00009
 \end{array}$$

$$\begin{array}{r}
 c_{10}=1 \\
 c_9=1 \\
 c_8=1 \\
 c_7=0 \\
 c_6=1 \\
 c_5=1 \\
 c_4=1 \\
 c_3=0 \\
 c_2=0 \\
 c_1=0 \\
 c_0=0 \\
 \hline
 1000.0101 + \\
 111.01001 \\
 \hline
 1111.02011
 \end{array}$$